Modes and Chord Progressions in Equal Tunings

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1. Diatonic Scales

This paper deals with four equal tunings—those of nineteen, seventeen, sixteen, and fifteen notes—and is the result of a study whose purpose was to find and describe chord progressions and scales within the equal tunings of thirteen through twenty-four notes in which one can perceive tonal functions, such as subdominants, dominants, or tonics. To be sure, all these tunings produce intervals that are extremely discordant; and in some, there are no consonances whatever. But even the most discordant tunings contain at least one modal substructure that is harmonically coherent; and this, along with the particular ease of modulation afforded by equal tunings only, makes such tunings especially interesting regarding the possibility of new musical

styles based upon forms in which thematic elements are set apart by different keys or modes.

When investigating a tuning for which there is little or no repertoire or tradition, it will be helpful to look for similarities between the new tuning and the familiar twelve-note equal tuning. As is well known, twelve-note equal tuning furnishes acceptably consonant major triads which may be treated as subdominant, dominant, and tonic within a major key, and rearranged by octave transpositions into an asymmetric array of major and minor seconds. If the tonic is C, and if we call a major second w and a minor second h, the familiar arrangement is as in Example 1.

C		D		E		F		G		A		B		C
	w		w		h		w		w		w		h	

EXAMPLE 1

The choice of the letters w and h follows from the habit of calling a major second a "whole step" and a minor second a "half step." But this establishes the special relation that the major second is exactly twice the size of the minor second, and this is not the case with other versions of the diatonic scale that have been used through music history. For example, both Pythagorean tuning and meantone tuning exhibit the same succession of w's and h's within one octave, but in neither case is a major second exactly twice a minor second.[1] It thus appears that twelve-note equal tuning, Pythagorean tuning, and meantone tuning might be regarded as particular members of a general family of diatonic tunings, which can be defined in the most general sense as the ascending succession of major and minor seconds shown in Example 1. In every case, the five w's and two h's add to one octave, or 1200 cents exactly; and if we call an octave a (so that $a = 1200$), we always have $5w+ 2h=a$. But w and h cannot be any numbers whatever satisfying this relation; for the scale to be ascending, we must have $w>0$ and $h>0$, and for the major second to be greater than the minor second, we must have $w>h$. This shows that for every particular member of the family, as described above, we must have $5w+2h=a$, and also $0<h<w$.

2. Equal Tunings That Contain Diatonic Scales

In the familiar twelve-note equal tuning, each of the five major seconds spans two chromatic degrees, requiring ten in all, while each minor second spans

two chromatic degrees, for a total of twelve. Now imagine the situation in which the major seconds are "whole steps" and the minor seconds are "third steps." In this case, each w will span three chromatic degrees and each h will span one, and so the five w's will require a total of fifteen chromatic degrees with two more needed for the h's—a total of seventeen notes within one octave. Thus we see that seventeen-note equal tuning contains diatonic scales as defined, for $w = (3/17)a$ and $h = (1/17)a$; and it is true both that $5w + 2h = a$ and $0<h<w$.

The general principle is as follows: If w spans x chromatic degrees and h spans y chromatic degrees, and if $0<y<x$, then the diatonic scale may be found among the notes of an equal tuning of $5x + 2y$ notes, where x and y are integers.

As another example, consider the diatonic scale consisting of five "whole steps" and two "quarter steps"; in this case, $x = 4$ and $y = 1$, and the number of notes in the corresponding equal tuning is $(5)(4) + (2)(1) = 22$. In a diatonic scale consisting of five "whole steps" and two "two-thirds steps," we have $x = 3$, $y = 2$, and $5x + 2y = 19$. And also if $x = 4$ and $y = 2$, then $5x + 2y = 24$. Note however that if $2< y< x$, then $5x + 2y>24$, and hence all such tunings are outside the range of this study. We are thus especially interested in the diatonic scales associated with the equal tunings of seventeen, nineteen, twenty-two, and twenty-four notes, and we begin with the tunings of nineteen notes and seventeen notes.

3. Diatonic Behavior Associated with Nineteen-Note Equal Tuning

As we have just seen, nineteen-note equal tuning contains diatonic scales in which a major second spans three chromatic degrees, while a minor second spans two. A convenient visualization of this scale may be found if we first write out the integers from zero through nineteen, and regard these as positions within the equal tuning occupied by the various notes. If we place middle C in position 0, and the C an octave above in position 19, we may locate the other notes of the scale by advancing three positions for each w and two positions for each h, as shown in Example 2.

positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
notes	C			D			E		F			G			A			B		C

EXAMPLE 2

From this, we see at once that a perfect fifth, such as CG, is equal to $(^{11}/_{19})a$ = 695.737 cents, smaller than pure (701.955 cents) by 7.218 cents. When played alone and sustained, a nineteen-note fifth sounds distinctly more out of tune than a twelve-note fifth of 700 cents, but not in a disagreeable way. Example 2 also shows at once that a nineteen-note major third is equal to $(^{6}/_{19})a$ = 378.947 cents, smaller than pure (386.314 cents) by 7.366 cents. Although this interval is more nearly pure than the twelve-note representation (400 cents—larger than pure by 13.686 cents), it seems very small to a trained musician, whereas one does not perceive the twelve-note version as extra large. The ear seems rather more willing to accept a larger third than a smaller third that is impure to the same degree—at least in my experience. From Example 2, we also see that a nineteen-note minor third (such as EG) is equal to $(^{5}/_{19})a$ = 315.789 cents, larger than pure (315.641 cents) by only .148 cents—an impurity that can be detected only under laboratory conditions. Thus a nineteen-note triad consists of intervals that depart from the pure versions by no more than 8 cents, as opposed to nearly 14 cents in the case of the twelve-note representation. I think it is safe to say that a nineteen-note major triad is smoother than its twelve-note counterpart; yet I cannot escape the sensation that the third of the nineteen-note version is flat. Much listening over many years has not changed my opinion on this point.

The nineteen-note major and minor seconds are equal to $(^{3}/_{19})a$ = 189.474 cents, and $(^{2}/_{19})a$ = 126.316 cents, respectively. This latter representation is the most troublesome of all the diatonic intervals, for it is the interval separating the leading tone from the tonic, and is quite perceptively larger than the twelve-note version. This, in combination with the small major third formed by the dominant and the leading tone, makes the leading tone seem very flat to a trained musician. This low leading tone is the most practically problematic aspect of the nineteen-note diatonic scale, for it causes a peculiarly tense expressive force to be associated with melodic minor seconds—a feature that must be taken into account when composing diatonic melodies.

Seventh chords associated with nineteen-note tuning are generally rather more discordant than the corresponding twelve-note versions. This stems primarily from the rougher nineteen-note versions of major seconds, minor seconds, and diminished fifths of $(^{10}/_{19})a$ = 631.579 cents. To my ear, the greater discordance of these unstable harmonies does not require any special treatment.

In sum, all diatonic progressions of triads and seventh chords have the same behavior and produce the same musical effect in twelve-note and nineteen-note tuning, save for slight differences only, the most noticeable being the peculiar tuning of the nineteen-note major scale.

4. The Family of Nineteen Major Keys and Their Notation

Clearly an equal tuning that contains one diatonic scale as defined must contain as many scales as notes in the tuning, each having the same intonation save for differences in register. In the present case, there must be nineteen distinct keys, each making the succession w, w, h, w, w, w, h within one octave, where $w = (\frac{3}{19})a$ and $h = (\frac{2}{19})a$. From this, we may assign F# to its proper position within the tuning relative to C in position 0. We first note that F is in position 8, and G is in position 11 (Example 2, Section 3). Since F#G must be a minor second of $(\frac{2}{19})a$, we see at once that F# is in position 9. Since F is in position 8, it appears that a sharp moves a note one position higher. By an extension of the same principle, a double sharp moves a note two positions higher, thus placing Fx in position 10.

To locate the flats, we observe that A is in position 14 and B is in position 17. Since AB♭ must be a minor second of $(\frac{2}{19})a$, it follows that B♭ is in position 16. More generally, the addition of a flat moves a note one position lower, and a double flat moves a note two positions lower. We may now extend the diagram of Example 2 to include all the notes from D♭♭ through Ax, and the result is as in Example 3.

positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
notes	C	C#	D♭	D	D#	E♭	E	E#	F	F#	G♭	G	G#	A♭	A	A#	B♭	B	B#	C
		D♭♭	Cx		E♭♭	Dx		F♭		G♭♭	Fx		A♭♭	Gx		B♭♭	Ax		C♭	

EXAMPLE 3

In many cases, the note in a given position admits of two different names, and this illustrates an array of nineteen-note enharmonic equivalents. In each case, the two equivalent notes, such as E# and F♭, or B# and C♭, represent a closing of a circle of nineteen fifths, as illustrated in Example 4.

19

F♭ C♭ G♭ D♭ A♭ E♭ B♭ F C G D A E B F# C# G# D# A# E# B#

19

EXAMPLE 4

The nineteen-note equivalents may be deduced at once from the corresponding twelve-note equivalents, simply by adding one more sharp, or one

more flat. For example, in twelve-note tuning, we have C#≡D♭;[2] hence in nineteen-note tuning, it is true both that C#≡D𝄫 (position 1), and Cx≡D♭ (position 2).

The notation of some keys requires a rather inconvenient arrangement of double sharps or double flats. For example, the scale whose tonic is in position 15 may be written A#, B#, Cx, D#, E#, Fx, Gx, A#, or B𝄫, C♭, D♭, E𝄫, F♭, G♭, A♭, B𝄫; there is no simpler way that is consistent with the arrangement of major and minor seconds within an octave.

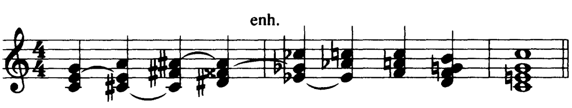
We observe that the two notes that triply divide each major second form a diminished second, such as C#D♭. As musicians would say, "the flats are higher than the sharps," in each case by $(1/19)a$ = 63.158 cents—a situation that seems at first perplexing, especially to string players and singers, but one that can be quickly adjusted to.

5. The Circle of Nineteen Fifths

Although diatonic relations are the same in twelve-note and nineteen-note tunings, the same is not true regarding chromatic progressions, owing largely to the closed circle of nineteen fifths, as opposed to twelve. As an illustration, consider a twelve-note modulating sequence in which each transposed repetition is lower than the one preceding by a minor third. If the initial tonic is C, the subsequent tonics are A, F#≡G♭, E♭, and C; thus the initial tonic returns at the fourth transposed repetition. In nineteen-note tuning, however, the initial tonic is not found until the nineteenth transposed repetition. The succession of tonics will be C, A, F#, D#≡E𝄫, C♭, A♭, F, D, B, G#, E#≡F♭, D♭, B♭, G, E, C#, A#≡B𝄫, G♭, E♭, and C. The effect is rather confounding, but not disagreeable.

In the above arrangement, we note that the seventh tonic is F; this establishes a new special relation connecting a tonic (C) and its subdominant (F). The progression in Example 5 illustrates the connection within a standard phrase. The progression is rather startling, although in a pleasing way.

It should be noted that in nineteen-note tuning, all modulating sequences, whatever their nature, must undergo nineteen transposed repetitions before the initial harmony returns. More generally, in any equal tuning in which the



EXAMPLE 5

number of notes is prime, all modulating sequences must undergo as many transposed repetitions as there are notes in the tuning before the initial harmony returns.

It is possible in many ways to close a circle of nineteen fifths after only a few modulations; all that is needed is for the sum of the number of positions on the circle of fifths associated with each tonic motion to be 19. For example, modulations that add three sharps, four sharps, or five sharps move a tonic down a minor third, up a major third, or down a minor second, respectively. As an example, we have 3+3+5+4+4=19; if the initial tonic is C, the successive tonics are A, F#, E#≡F♭, A♭, and C once again. If the modulations proceed at a slow pace, the effect is subtle.[3]

6. Chromatic Intervals and Progressions

Many nineteen-note chromatic progressions will bring about alien melodic intervals—in particular, $(1/19)a$ = 63.158 cents, construed as a chromatic semitone or a diminished second; and $(4/19)a$ = 252.632 cents, construed as an augmented second or a diminished third. To a trained musician, these intervals may or may not be disturbing in a given context; in this regard, subjective judgment seems to be the best guide. Note that the superior part in Example 5 contains two melodic intervals equal to $(1/19)a$, namely AA# and C♭C♮.

An especially intriguing use melodically for $(4/19)a$ places this interval in a context where it is perceived as one-half of a perfect fourth, for an equally divided perfect fourth is outside the experience of a musician with twelve-note habits. Closely related is the enharmonic equivalence of a diminished seventh and an augmented sixth, such as CB𝄫 and CA#, which leads to a remarkable and expressive chromatic progression connecting a tonic and its dominant. Imagine first a C major chord followed by a German sixth in that key, tuned A♭CE♭F#. Next construe augmented sixth A♭F# as a diminished seventh, by replacing A♭ by its enharmonic equivalent Gx. Now treat diminished seventh GxF# as the third and ninth in a minor dominant ninth chord whose root is E#, and resolve it in the usual way—i.e., both parts by minor second $(2/19a)$ in contrary motion to A#E#, and complete this major triad by adding a Cx. Note that A# is four positions lower than C (Example 3, Section 4); thus if the same progression is repeated, but transposed down by $(4/19)a$, the next triad will be four positions lower than A#≡B𝄫, namely G. We may then treat G as the dominant of C, and make a normal cadence. The progression is written as shown in Example 6. In this progression, the A# triad is heard as lying midway between the triads whose roots are C and G. To my ear, the effect is strikingly pleasant, but very strange indeed.

EXAMPLE 6

The chromatic progressions illustrated do not by any means exhaust the subject; however they illustrate how chromatic behavior may be quite different in twelve-note and nineteen-note tuning. The contrast between diatonic and chromatic elements in nineteen-note tuning is rather more than what we are accustomed to, and in particular, chromatic melodies are likely to sound especially alien. On the other hand, modulations that close a circle of twelve fifths or nineteen fifths produce a remarkably similar effect.

All this suggests that a substantial enrichment of the tonal repertoire is possible within the medium of nineteen-note equal tuning. If the full resources of the tuning are utilized, the style that is likely to emerge will be similar to that of the late nineteenth century. But I cannot find any standard work that sounds better in nineteen-note than in twelve-note tuning, whether nearly diatonic, or highly chromatic. The chance that nineteen-note tuning will replace twelve-note tuning for any portion of the existing repertoire is, in my judgment, exactly nil. While it is theoretically possible to instruct string, wind, and brass players how to play in nineteen-note equal tuning, I am persuaded that this is as yet far down the road, and may never come. Special keyboard instruments, or fretted stringed instruments specifically designed for the tuning would be more appropriate. At present, the only practical means of realizing nineteen-note compositions is electronic synthesis.

7. Diatonic Behavior Associated with Seventeen-Note Equal Tuning

As was shown earlier (Section 1), seventeen-note equal tuning contains diatonic scales in which a major second spans three chromatic degrees, while a minor second spans one. We may locate the notes forming C major relative to the seventeen equal chromatic degrees by the same method that was used to locate the scale in nineteen-note tuning, and we obtain Example 7.

positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
notes	C			D			E	F			G			A			B	C

EXAMPLE 7

From this, we see at once that a perfect fifth, such as CG, is equal to $(^{10}/_{17})\boldsymbol{a}$ = 705.882 cents, larger than pure by 3.927 cents—a better representation than what is found in nineteen-note tuning. Example 7 also shows that a major third is equal to $(^{6}/_{17})\boldsymbol{a}$ = 423.529 cents. This is larger than pure (386.314 cents) by 37.216 cents, larger than Pythagorean (407.820 cents) by 15.709 cents, and smaller than the wolves of meantone tuning (427.373 cents) by only 3.843 cents. In consequence, it is a jangling discord, and altogether inappropriate in a consonant harmony unless concealed by distribution and timbre. The same may be said only a little less vehemently regarding a seventeen-note minor third of 282.353 cents—smaller than pure (315.641 cents) by 33.288 cents.

The subjective discordance of a seventeen-note major third is greatly intensified when it is combined with a fifth to make a root-position major triad. This "harmony" in consequence seems exceedingly harsh—so much so that I can find virtually no use for it in any tonal context, much less as a final chord in a cadence. Hence a seventeen-note harmony rule that may seem strange to a trained musician—root-position major triads should be avoided in any context whatever. This opinion is the result of much listening which has not conditioned my ear to accept the discordance of seventeen-note triads. Diatonic seventh chords are generally more satisfactory than triads, for the major or minor seventh tends to mask the discordance of the thirds. Even so, all distributions are more piquantly discordant than their twelve-note counterparts. This is not unduly disturbing in dominant sevenths and half-diminished sevenths, for these dissonant harmonies are unstable regardless of the level of discordance. To my ear, the most satisfactory harmony at the close of a cadence is a root-position minor seventh chord; another possibility is a minor triad with an added major sixth. This suggests that the most useful diatonic arrangements might be the Dorian, Phrygian, and Aeolian modes. All modal sequences or progressions of seventh chords are satisfactory, subject to the restrictions regarding the final harmony in the phrase. To be sure, such progressions exist also in twelve-note tuning, and in each case, the twelve-note version is less discordant than the seventeen-note version. But diatonic seventh chords exhibit a different character in the two tunings—the seventeen-note versions impart a noticeably zestier and livelier quality to modal progressions, and this may be turned to an advantage by suitable choice of tempo, timbre, distribution, and rhythmic arrangement.

8. The Family of Seventeen Diatonic Scales and Their Notation

We may add the names of the notes missing from Example 7 (Section 7) by using the same means as that employed during the discussion of nineteen-

note tuning (Section 4). We first note that F is in position 7, and G is in position 10. Since F#G must be a minor second of $(\frac{1}{17})\boldsymbol{a}$, we see at once that F# is in position 9; this shows that a sharp moves a note two positions higher, and also that a double sharp moves a note four positions higher. We next note that A is in position 13 and B is in position 16; since AB♭ must be a minor second of $(\frac{1}{17})\boldsymbol{a}$, it follows that B♭ is in position 14. More generally, a flat moves a note two positions lower, and a double flat moves a note four positions lower. We may now extend the diagram of Example 7 to include all the notes from A𝄫 through G𝄫 (Example 8).

positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
notes	C	D♭	C#	D	E♭	D#	E	F	G♭	F#	G	A♭	G#	A	B♭	A#	B	C
		B#	E𝄫		Cx	F♭			E#	A𝄫		Fx	B𝄫		Gx	C♭		

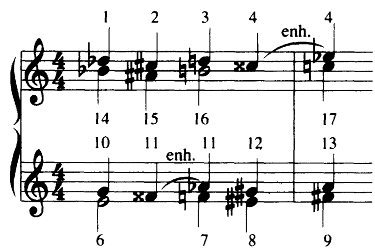
EXAMPLE 8

For seventeen-note tuning, yet a new set of habits must be learned regarding notes that are enharmonic, such as D♭ and B#, and unfortunately seventeen-note enharmonics are not so readily deduced from twelve-note enharmonics as are nineteen-note enharmonics. In my experience, however, this is not a lasting problem. Once again, notes that we are accustomed to regarding as the same, such as C# and D♭, are distinct, and divide major second CD into three equal parts. But in the present case, the ascending succession is C, D♭, C#, D, whereas in nineteen-note tuning, the ascending succession is C, C#, D♭, D, i.e., the order of C# and D♭ is reversed.

9. Chromatic Intervals and Progressions

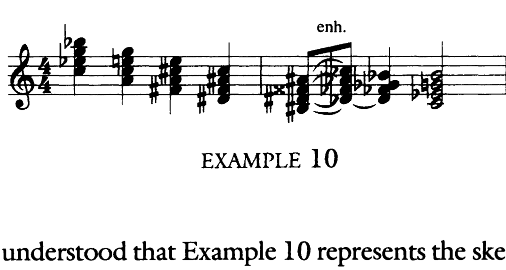
Problems are associated with $(\frac{2}{17})\boldsymbol{a}=141.176$ cents, whether construed as a chromatic semitone or a diminished third. It is intensely discordant, and when taken out of context, sounds like either a large minor second, or a small major second. To my ear, this ambiguity is rather disturbing, and I think that $(\frac{2}{17})\boldsymbol{a}$ should be used only sparingly in a melody. The same problem is exhibited by any interval that is similarly ambiguous, the awkward range being from about 135 cents to 160 cents in round numbers. It is also my opinion that $(\frac{2}{17})\boldsymbol{a}$ is especially disturbing in contexts where it is perceived as being one-half of a minor third. This sharply reduces the number of agreeable chromatic progressions, for there are very few such that do not bring about melodic motion by a chromatic semitone. However we may find agreeable chromatic progressions made of diminished-seventh chords, in which all parts move by $(\frac{1}{17})\boldsymbol{a}$,

and Example 9 is representative. Numbers give positions of the notes according to the diagram of Example 8 (Section 8). The progression may be satisfactorily continued as a modulating sequence, and is also effective if placed over a G pedal.



EXAMPLE 9

Since 17 is prime, all modulating sequences in seventeen-note tuning must undergo seventeen transposed repetitions before the initial harmony returns.[4] However there are many ways to close a circle of seventeen fifths with only a few modulations. An especially smooth modulation in seventeen-note tuning connects two minor-seventh tonics whose roots differ by a minor third, such as CE♭GB♭ and ACEG. It is only necessary to follow one tonic directly with the other, while individual parts move no more than $(2/17)a$. However, chromatic semitone E♭E♮ is better not placed in a prominent melodic part. If similar modulations are repeated, each tonic is lower than the preceding by a minor third of $(4/17)a$. Thus the tonic at the fourth repetition is lower than the initial tonic by $(16/17)a$, and the circle may be closed by yet another modulation down by $(1/17)a$. Example 10 presents an outline of such a progression.



EXAMPLE 10

It should be understood that Example 10 represents the skeleton of a more extensive progression or phrase in which all the minor-seventh chords are

treated as tonics in the Dorian, Phrygian, or Aeolian modes. This arrangement cannot be realistically approximated in twelve-note tuning, and yet its alien aspect is so subtle that trained musicians are seldom conscious of it unless it is pointed out to them.

One of the surprising features of seventeen-note tuning is how readily the rough aspect of discordant versions of diatonic harmonies—mainly seventh chords—can be effectively masked by appropriate choices of distribution, timbre, rhythm, and tempo. The tuning is better suited to pieces in a quick tempo, and could certainly generate further repertoire. It might have some applications to commercial music or advertising, owing to its optimistic, "upbeat" nature.

10. Distinction between Dissonance and Discordance

By now, the reader must have noticed that I draw a distinction between "dissonance" and "discordance" that is not generally observed by other writers on music theory. For example, major and minor thirds are described as "jangling discords" (Section 7), but not dissonances. On the other hand, it is stated that dissonant harmonies are unstable regardless of the level of discordance. The distinction is between an interval or chord that has a rough sound, such as a major-seventh chord, which is discordant in any tuning, and a chord that contains a combination of tendency notes, such as a dominant seventh, a diminished seventh, or an augmented sixth; these latter are dissonant whatever the tuning. The reason for the distinction is that discordant tunings of triads do not render the triads dissonant as defined above. To my ear, a discordant tuning of a harmony that is not dissonant is not an agreeable sound, but considerable discordance is acceptable in the case of a dissonant harmony, and may even enhance it; but even here, there is a limit. At present, I am persuaded that the level of dissonance associated with a particular harmony, such as a diminished-seventh chord, is only minimally affected by its degree of discordance. Even a dominant-seventh chord in pure tuning, i.e., in frequency ratios 4 : 5 : 6 : 7, which is not discordant in the least, retains the instability that is normally associated with this chord in tonal circumstances. It thus appears that dissonance and discordance are not only separate phenomena, they tend to be independent. The distinction is not revealed by the standard twelve-note equal tuning, for triads are acceptably consonant, while dissonances are slightly discordant. In fact, twelve-note tuning contains no intervals whatever that could be accurately described as "jangling discords." However the distinction between dissonance and discordance exists to some extent in all the tunings that will eventually be covered by this study.

11. Equal Tunings That Contain Octatonic Modes

An important modal substructure of twelve-note equal tuning is an array of notes in which whole steps and half steps alternate, such as C, D, E♭, F, F#, G#, A, B, C. If we start at C and take every other note from this array, we obtain diminished-seventh chord CE♭F#A, while the remaining notes make diminished-seventh chord DFG#B. In a close-position diminished-seventh chord, all the adjacent intervals are equal to 300 cents, and hence a twelve-note diminished-seventh chord may be regarded as a division of an octave into four equal parts. This suggests that octatonic modes may be found in any equal tuning in which the number of notes is divisible by four; in addition, intervals of 300 cents must be unequally subdivided. It can thus be seen that sixteen-note equal tuning will contain families of octatonic modes; the same is true for twenty notes, twenty-four notes, etc. In twelve-note tuning, there are only three distinct diminished-seventh chords, for if a diminished-seventh chord is transposed up or down by 300 cents, the result is the same notes, save for enharmonic differences. It thus appears that there are three distinct octatonic modes in twelve-note tuning, each formed by taking a different pair of diminished-seventh chords. Thus in addition to the mode containing C and D, there is a transposition that contains C and C#, and yet another that contains C# and D. The symmetry associated with each octatonic scale assures that any harmony within it will exist in transpositions that are successively higher by 300 cents.

Longer works written at the beginning of the twentieth century may exhibit substantial passages that are entirely within a single octatonic mode. Stravinsky's *Le Sacre du Printemps* (1912) is a familiar example—between rehearsal numbers 42 and 43, we find the mode that contains C and C#; this mode is used again between numbers 193 and 195. A work making extensive use of octatonic passages is the Sonata for Violin and Piano by Ernest Bloch (1922). Between rehearsal numbers 5 and 6, we find the mode that includes C and D, and the same mode is used exclusively from five bars after number 6 through three bars after number 7. From eight bars before number 13 through three bars after number 13, we find the mode that includes C and C#; and the mode that includes C# and D is used exclusively between thirteen bars before number 21 through five bars after number 22. All these passages suggest meditation, religiosity, agitation, or mystery.

The highly characteristic expression associated with a single octatonic scale is not compromised by the use of occasional passing notes in melodies, or by an extraneous note in a complex harmony of at least six notes.

In general, a succession of three complex dissonant chords, one in each of the three different octatonic modes, produces a sensation of coherent harmonic motion that is not unlike subdominant to dominant to tonic. But in the absence of a diatonic stabilizing element, one cannot be sure where the

tonic is. The effect produced is great agitation, especially in a rapid tempo. Once again, *Le Sacre du Printemps* affords a representative example; the progression shown in Example 11 occurs seven times during the first three pages of the Danse Sacrale. The first chord is in the mode that includes C and C#; the second is in the mode that includes C# and D (B# is extraneous), and the third is in the mode that includes C and D (C# is extraneous).

EXAMPLE 11

If a progression such as that of Example 11 is played very slowly and embellished, one perceives each change of harmony as a modulation, rather than as a different function within a tonal framework. As an example, see Scriabin's Seventh Piano Sonata, Op. 64 (1911–1912). Within the first nine bars, we move from the mode that includes C and C# to the one including C and D, then to the one including C# and D, and then return to the original. There are a few extraneous notes, used as expressive inflections.

12. Octatonic Modes Associated with Sixteen-Note Equal Tuning

In sixteen-note tuning, a minor third spans four chromatic degrees, and is thus equal to $(4/16)a = (1/4)a = 300$ cents. The unequal subdivision of a minor third may be achieved by $(1/16)a$ and $(3/16)a$, or vice versa. We find that $(1/16)a$ = 75 cents, and $(3/16)a = 225$ cents. This latter interval seems rather large for a major second, although not unacceptably so. We thus find a sixteen-note octatonic scale in positions 0, 1, 4, 5, 8, 9, 12, 13, and 16. In addition, we may find transpositions of the same mode, starting in positions 1, 2, and 3. However the mode starting in position 4 duplicates the initial mode; hence sixteen-note tuning contains four distinct octatonic modes, as opposed to three in twelve-note tuning.

We next determine which harmonies exist within a single octatonic mode in acceptable versions. We may find a recognizable major triad in positions 0, 5, and 9; the major third is represented by $(5/16)a = 375$ cents, and the perfect

fifth by $(9/16)a$ = 675 cents. The major third is smaller than pure (386.314 cents) by 11.314 cents—less than the impurity associated with a Pythagorean major third (407.820 cents, larger than pure by 21.506 cents). More troublesome is the perfect fifth, which is smaller than pure (701.955 cents) by 26.955 cents. As we have seen, the third and fifth form a minor third of 300 cents, as in twelve-note tuning; but in sixteen-note tuning, both notes are lower than their twelve-note counterparts by 25 cents. In consequence, the third and fifth of a sixteen-note triad sound very flat—sufficiently so to render the triad unacceptable, as least to my ear. Minor triads are generally less sensitive to tuning irregularities than major triads; in the present case, however, I think the same restriction should be applied to both. In first inversion, however, the third in the lowest part seems to soften the effect of the fifth or fourth to such an extent that the chord is usable in any distribution.

It will be noted that the limitations placed on sixteen-note triads are less restrictive than what is associated with the seventeen-note versions (Section 7). The reason is that the shrillness associated with the enlarged seventeen-note major third is not present in the case of the small sixteen-note perfect fifth. To be sure, both triads sound wrong in nearly equal measure; but the seventeen-note version exhibits a biting edge that is not present in the sixteen-note triad, and hence the distinction.

We find a minor seventh chord in positions 0, 4, 9, and 13; the interval between the fifth and seventh is a minor third of $(4/16)a$ = 300 cents, the third and seventh form a perfect fifth of $(9/16)a$ = 675 cents, while the root and seventh are distant by $(13/16)a$ = 975 cents. This is an acceptable representation of a minor seventh, which is larger than the semipure interval whose ratio is $7/4$ (968.826 cents) by only 6.174 cents. As with seventeen-note tuning, the added seventh tends to mask the discordance of the underlying triad to the extent that a minor-seventh chord is satisfactory at the close of a cadence. We may find a dominant seventh in positions 0, 5, 9, and 13; this acceptable version may be rendered less discordant by omitting its fifth (position 9). An acceptable half-diminished seventh is found in positions 0, 4, 8, and 13; an improved version results upon leaving out the note in position 4, thereby eliminating the perfect fifth between the notes in positions 4 and 13. Also acceptable are minor dominant ninths and altered chords featuring lowered fifths, as well as more complex dissonant harmonies. Note that all the acceptable harmonies are dissonant, with the exception of a minor-seventh chord, suggesting that this latter harmony should be preferred as the most stable within the tuning.

13. Notation of Sixteen-Note Equal Tuning

We now come face to face with the problem of devising a comprehensible notation for sixteen-note equal tuning. It will be recalled that in the case of

nineteen notes and seventeen notes, this was neatly accomplished by using a general principle associated with diatonic scales (Sections 4 and 7). But the principle is inapplicable to sixteen-note tuning, which contains no diatonic scales. What is wanted is a notation that is compatible with the existing notation, and with musical habits. To meet these requirements, I think it essential that the five-line staff should be preserved, and that octaves should have the same appearance in all the equal tunings. In addition, the usual direction and approximate distance associated with sharps and flats should be retained. Since sixteen-note tuning may be viewed as a combination of four intertwined diminished-seventh chords that may be combined into octatonic scales, it is highly desirable that the notation of these chords should have the appearance of conventional diminished-seventh chords. Regarding the recognizable but very discordant sixteen-note triads, either the usual spelling or some enharmonic rearrangement would be acceptable. I must confess that the requirements of both triads and diminished-seventh chords have not suggested any approach other than trial and error. The satisfactory scheme finally arrived at is shown in Example 12; if there is a logical way to find it, I should be most interested. The four enharmonics needed—positions 1, 5, 9, and 13—are identical to those found in nineteen-note tuning (Section 4), and are easily learned by a musician with twelve-note habits.

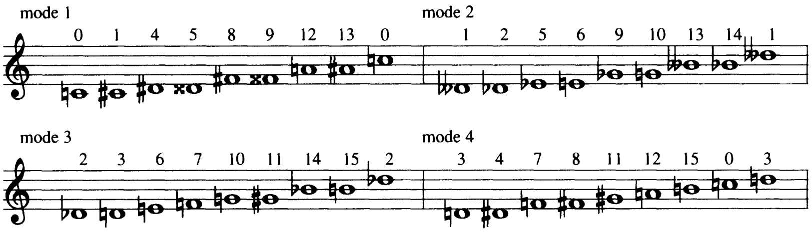
positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
notes	C	C#	D♭	D	D#	Dx	E	F	F#	Fx	G	G#	A	A#	B♭	B	C
		D♭♭				E♭				G♭				B♭♭			

EXAMPLE 12

We may write out the octatonic modes starting with the notes in positions 2 and 3, or position 3 and 4, simply by referring to Example 12. Regarding the other modes, there are choices to be made, for both contain the notes in positions 1, 5, 9, and 13—notes for which there are two possible names. The preferred spellings are shown in Example 13.

To a trained musician, each of the four modes has the same appearance as in twelve-note tuning, with a few unusual enharmonic spellings. The principal disadvantage of the notation is the different enharmonic spellings of various transpositions of the same harmony. For example, the four minor-seventh chords found in mode 1 are written CD# FxA#, D#F#A#C#, F#AC#Dx, and ACDxFx.

The harmonic relations associated with any one of the octatonic modes are exactly the same in twelve-note and sixteen-note tuning; the sole difference is that the sixteen-note version is noticeably out of tune, imparting a peculiar

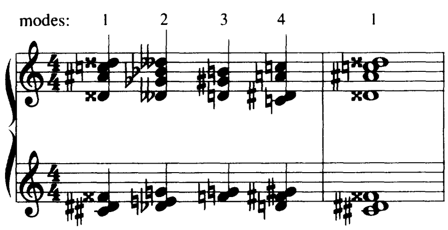


EXAMPLE 13

tension that is not present in the twelve-note version. For this reason, lengthy passages in a single octatonic mode do not exploit sixteen-note tuning in any advantageous way.

14. Cyclic Progressions Involving the Four Octatonic Modes

We may find a variety of progressions that produce much the same effect as the fragment quoted earlier from *Le Sacre du Printemps* (Example 11, Section 11). Example 14 is representative. The octatonic modes are identified by numbers as in Example 13.



EXAMPLE 14

One is naturally led to ask next how many different successions of the four harmonies of Example 14 might make similarly coherent progressions. If the initial and final harmonies are retained, there are six possibilities to investigate, namely 1–2–3–4–1 (as above), also 1–2–4–3–1, 1–3–2–4–1, 1–3–4–2–1, 1–4–2–3–1, and 1–4–3–2–1. It may come as a surprise that all are equally viable; in fact, when played rapidly, it is difficult to be certain which one is being heard. To be sure, one notices that the successions 1–3, 3–1, 2–4, and

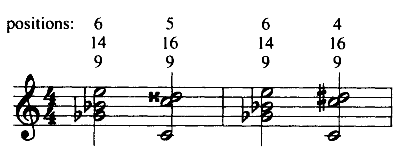
4–2 are rather more disjunct than the others. The reason is that modes 1 and 3 have no notes in common, and the same is true of modes 2 and 4. In all the other successions, the modes have four notes in common. This is a more intricate situation than what is found in twelve-note tuning, for its three modes may be arranged into two successions only, one being the other in reverse order.

Some care is needed regarding melodic intervals in chromatic progressions such as that of Example 14, for they may bring about melodic intervals equal to $(\tfrac{2}{16})a = 150$ cents. This ambiguous interval is disturbing in much the same manner as $(\tfrac{2}{17})a$ (Section 9), especially when placed in the superior part. In Example 14, we find $(\tfrac{2}{16})a$ in the highest part between D𝄫 and B♮ descending; in this context, the harmonic surprise tends to mask the awkward melodic contour.

If progressions similar to the one of Example 14 are expanded so that there are several chords within each mode, then each change of mode is heard as a modulation. The same phenomenon was observed with respect to the three octatonic modes contained by twelve-note tuning (Section 11). The four modes may then be used to delineate sections within larger musical forms.

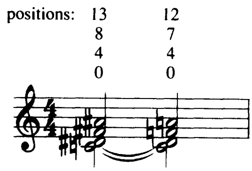
15. Diatonic Progressions and Plagal Cadences

Attempts to arrange the diatonic seventh chords into more conventional progressions bring about difficulties, for most such progressions involve $(\tfrac{2}{16})a$ as a melodic interval, construed either as a minor second or a major second. This problem comes up in the basic progression V^7–I in major or minor. If both chords are presented without fifths, thereby eliminating the most discordant interval, and the root of the resolution is C, we have the situation in Example 15. In both cases, roots fall by $(\tfrac{9}{16})a$ = 675 cents, and the departure from the leading tone is by $(\tfrac{2}{16})a$. When the resolution is to a major triad, its third is approached by $(\tfrac{1}{16})a$; the progression thus brings about two different melodic minor seconds, one of which is twice the size of the other. When the resolution is to a minor triad, the superior part descends by $(\tfrac{2}{16})a$—in this case construed as a major second. This demonstrates the ambiguity associated with $(\tfrac{2}{16})a$—a feature that I have not learned to like. Hence I think the progression V^7–I should be avoided in all contexts.



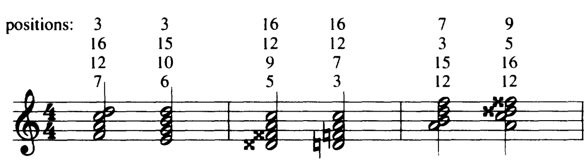
EXAMPLE 15

There seem to be very few ways to establish a tonic by using common chords that avoid $(2/16)a$ as a melodic interval. As an example, we have the progression II^7–V^4_3 in minor (Example 16). Aside from noticeable differences of intonation, the progression produces the same effect as its twelve-note analog, which may be played on a piano as written. There can be no doubt that the progression establishes B♭ as a tonic; but as we have just seen, there is no satisfactory way to resolve the dominant to a tonic harmony.



EXAMPLE 16

Other diatonic progressions that have proven useful are shown in Example 17. Such progressions may occasionally serve as plagal cadences; they are also satisfactory when played in reverse order. However, all bring about the possibility of melodic intervals equal to $(2/16)a$, and thus require especial care regarding distribution and voice leading.



EXAMPLE 17

16. Modulating Sequences Made of Diatonic Elements

Progressions like those of Example 17 may be arranged into a variety of pleasing modulating sequences in which the interval of transposition is a minor third of $(4/16)a = 300$ cents. Example 18 is representative.

Note that the superior part uses only notes found in octatonic mode 1 (Example 13), thereby avoiding melodic intervals equal to $(2/16)a$. In fact, $(2/16)a$ is found only in the tenor part, where it is effectively concealed. This

and similar progressions have twelve-note analogs that may be played on a piano as written.

EXAMPLE 18

A different class of modulating sequences may be found where the interval of transposition is $(2/16)a = 150$ cents. These progressions have no twelve-note analogs, for an interval of 150 cents cannot be realistically approximated on a piano. Example 19 is based on the progression of Example 16. In this example, the two higher parts descend constantly by $(1/16)a$, while the two lower parts descend by $(2/16)a$. In my judgment, the sequence is not good in distributions where a succession of $(2/16)a$'s occurs in the superior part.

EXAMPLE 19

It thus appears that sixteen-note tuning contains very few diatonic elements, while most chromatic elements have their origin in the family of octatonic modes. Since these modes are so readily identifiable and narrowly expressive, sixteen-note tuning may be expected to impart substantial stylistic restrictions in practical situations. This, in combination with the melodic restrictions associated with $(2/16)a$, suggests that the tonal vocabulary is quite small. In view of this, I doubt that the tuning will generate a large or varied repertoire.

It can now be seen that sixteen-note tuning is in a totally different category from the tunings of nineteen and seventeen notes. But this is not so much a function of its level of discordance, for seventeen-note tuning is discordant to virtually the same degree. The difference is that the tunings of seventeen and nineteen notes are essentially diatonic, there being as many major keys as there

are notes in the tuning. On the other hand, sixteen-note tuning is essentially chromatic; in addition, there are only four distinct transpositions of its characteristic mode. It must be concluded that diatonic elements provide a greater musical variety than what is associated with combinations of octatonic modes.

17. The Acceptable Major and Minor Triads within Fifteen-Note Equal Tuning

Since fifteen is divisible by three, it is clear that fifteen-note equal tuning contains a division of an octave into three equal parts, as does any tuning in which the number of notes is divisible by three. Hence a fifteen-note augmented triad is identical to the corresponding harmony found in twelve-note tuning; in each case, adjacent intervals are equal to 400 cents. We next observe that $(4/15)a$ = 320 cents; this interval exceeds a pure minor third (315.641 cents) by only 4.359 cents. A fifteen-note major third thus spans five chromatic degrees, while a minor third spans four. We may thus find a recognizable major triad in positions 0, 5, and 9; this shows that a perfect fifth is represented by $(9/15)a$ = 720 cents. This is greater than a pure perfect fifth (701.955 cents) by 18.045 cents—an amount that many theorists find too large to be acceptable. However, the roughness associated with a 720-cent fifth may be concealed by an appropriate choice of timbre and distribution; one hears then only that the higher note is noticeably sharp. At present, much practical experience with 720-cent fifths brings me to disagree with the conventional wisdom that such fifths are unacceptably large. In my opinion, $(9/15)a$ is a satisfactory tuning for the root and fifth of a major or minor triad in any inversion.

18. Seventh Chords and Melodic Intervals

A major-seventh chord is found in positions 0, 5, 9, and 14; the third and seventh make a perfect fifth of $(9/15)a$, while the fifth and seventh make a major third of $(5/15)a$. The major seventh is represented by $(14/15)a$ = 1120 cents—a slightly large version of this fundamentally discordant interval that is not disturbing. Hence a major-seventh chord may be used in the usual way without reservation.

A minor-seventh chord is found in positions 0, 4, 9, and 13, thus presenting the minor seventh as 1040 cents—a considerably enlarged version. This in combination with the enlarged perfect fifths renders the harmony considerably more discordant than we are accustomed to, but not excessively so.

We may find a dominant seventh in positions 0, 5, 9, and 13; this representation is quite discordant, owing to the diminished fifth of $(8/15)a$ = 640 cents—a very large version—found between its third and seventh. Also to be considered is a version in which the seventh is in position 12; the third and seventh are now separated by $(7/15)a$ = 560 cents—also very discordant. In addition, the interval between the fifth and seventh is now equal to $(3/15)a$ = 240 cents—too small to be perceived as a true minor third. Even so, both versions have been found to have practical applications, but I can find no general principle suggesting the one or the other. The sole guide appears to be subjective judgment in a particular situation.

There are melodic difficulties associated with $(2/15)a$ =160 cents and $(3/15)a$ = 240 cents. The former is an extremely small representation of a major second, being at the high end of the awkward range described in Section 9, while the latter is large enough to be heard as an augmented second or minor third in some contexts. If a major third of $(5/15)a$ is outlined melodically by the ascending succession of $(3/15)a$ followed by $(2/15)a$, one hears very plainly the substantial difference (80 cents) between the two major seconds. The musical effect is not very pleasing, for it makes the second note seem extremely sharp. If the order of major seconds is reversed, the effect is worse, for it makes the second note sound exceedingly flat. Experience now shows that such contours cannot be entirely avoided without unacceptably reducing the vocabulary of the tuning.

19. Chromatic Dissonances

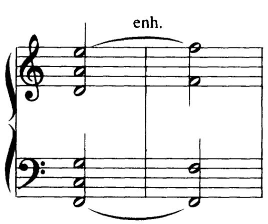
Within fifteen-note tuning, most chromatic dissonances are either excessively discordant, or sufficiently alien to be barely recognizable. In a close-position diminished-seventh chord, the three minor thirds are represented by $(4/15)a$, and the augmented second by $(3/15)a$. But wherever the smaller interval is placed, it sounds like a large major second, and not a minor third or an augmented second. In consequence, a fifteen-note diminished-seventh chord is not recognizable as such when taken out of context. I do not find myself wanting to use these chords within a framework of a tuning that contains acceptably consonant triads. On the other hand, a minor dominant ninth missing its fifth (positions 0, 5, 12, and 16) is satisfactory when treated in the usual manner.

20. Development of the Fifteen-Note Notation

Thus far, our discussion of fifteen-note triads and seventh chords strongly suggests that the principal harmonic units of the tuning are major and minor

triads, along with occasional use of major- and minor-seventh chords, and relatively infrequent use of dominant sevenths, dominant ninths, or chromatic dissonances. With this in mind, it is plain that the notation should present triads essentially as we are accustomed to seeing them—namely, a close-position triad should appear as a major third plus a minor third, with a perfect fifth between the two outer pitches.

A suggestion how to proceed comes from the fact that the sum of five 720-cent fifths is exactly equal to three octaves, as is evident from the relation $(5)(720) = (3)(1200) = 3600$. Now consider Example 20. This shows that any interval that appears to be a diatonic minor second is actually an enharmonic unison—a state of affairs that takes some getting used to. The ascending succession F, C, G, D, A, E thus represents a closed circle of five perfect fifths. Hence if the pitches forming the circle are rearranged by octave transpositions into an ascending succession within one octave, the result is a division of the octave into five equal parts.[5] We thus have C, D, E, G, A, C, in which each adjacent interval is equal to $(\frac{3}{15})a = 240$ cents, illustrating how this ambiguous interval may be construed as either a major second or a minor third. Since E≡F, the same arrangement may be written C, D, F, G, A, C; also possible is B, D, E, G, A, B, for B≡C. In general, what appears to be a pentatonic scale is actually a division of an octave into five equal parts.



EXAMPLE 20

So far, we have developed a notation for only one third of the notes of the tuning; notation of the additional ten notes may be achieved by the introduction of two new accidentals, which will appear as ↑ and ↓. The circles make it clear exactly which note in a chord is affected, by placing the accidental unmistakably on a line or in a space. I call these accidentals "up" and "down;" accordingly D↓ is read "D-down," F#↑ is read "F-sharp-up," and so on. Thus the symbol ↑ raises a note by 80 cents, whereas ↓ lowers a note by the same amount. Hence it is also true that C#↓≡D↓, also D↑≡E♭↑, etc. (The new accidentals will have applications to other tunings as well; in order to be as consistent as possible, they will always move a note by a relatively small

interval—i.e., never by more than 100 cents.) We may now write out a complete fifteen-note equal chromatic scale (Example 21).

positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
notes	C	C↑	D↓	D	D↑	E↓	E	F↑	F#↓	G	G↑	A↓	A	A↑	B↓	C
	B		C#↓		E♭↑		F				A♭↑		B♭	B♭↑		B

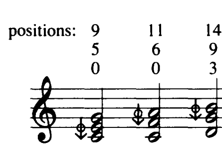
EXAMPLE 21

Enharmonics are chosen so as to make possible the notation of all fifteen major and minor triads as suggested at the beginning of this section. In the case of the first few major triads, we have C, E↓, G (positions 0, 5, and 9), C↑, E, G↑ (positions 1, 6, and 10), D↓, F↑, A↓ (positions 2, 7, and 11), and so on. The reader will find it useful to write out the remaining major and minor triads. Experience indicates that this notation is satisfactorily applicable to practical situations; however, rather more enharmonics than those shown in Example 21 will occasionally be needed.

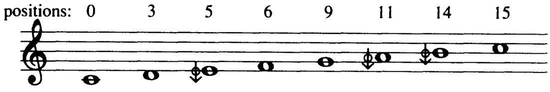
21. The Major Scale Generated by Rearranging the Notes Forming the Primary Triads

If the major triads whose roots are F, C, and G are treated as subdominant, dominant, and tonic, respectively, they may be arranged into a scale in which there exists a recognizable approximation of the progression I–IV–V–I. In close position, we have Example 22. The resulting scale is thus written in Example 23.

Note that CD, FG, and A↓B↓ are represented by $(3/15)a$, DE↓ and GA↓ by $(2/15)a$, while E↓F and B↓C are equal to $(1/15)a$. Subjectively, this scale sounds badly out of tune, for D seems very sharp, while A↓ seems equally flat. More light is thrown on this problem if we observe that DA↓ spans only eight



EXAMPLE 22



EXAMPLE 23

chromatic degrees; all the other perfect fifths span nine. In fact, DA‡ is exactly the same size as augmented fourth FB‡, namely 640 cents—an interval that cannot be construed as a perfect fifth. Hence DFA‡ is not a recognizable representation of the second-degree triad in C major. If D is replaced by D‡, the triad is tuned like the other minor triads; but now, D‡ must be followed directly by D♮ in the progression II–V. In other words, the root of II is 80 cents lower than the fifth of V; and when the two notes are brought side by side, the effect is grotesque and utterly unacceptable. Many hours of listening over many years have not changed my mind on this point. We thus have a fifteen-note rule that seems very strange to a musician trained in traditional harmony—the progression II–V in a major key must be totally avoided in any context.

There is also a severe restriction associated with the seventh-degree triad, for diminished triad B‡DF is so discordant in any inversion that it produces a shock in a diatonic context. This also effectively eliminates all uses of the second-degree triad in minor. The problems associated with II, II–V, and VII are very restrictive indeed; for there are very few extended progressions that avoid the troublesome degree successions.

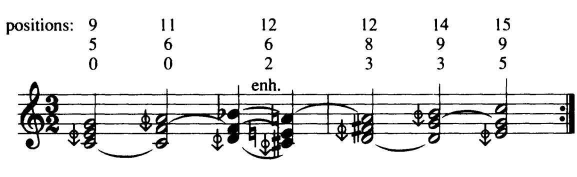
It is useful to compare the scale of Example 23 with the one found in just tuning.[6] In just tuning, CD, FG, and AB are major tones of 203.910 cents (ratio $9/8$), DE and GA are minor tones of 182.404 cents (ratio $10/9$, while EF and BC are diatonic semitones of 111.731 cents (ratio $16/15$). Thus we see that $(3/15)a$ corresponds to a major tone, $(2/15)a$ to a minor tone, and $(1/15)a$ to a diatonic semitone. But the approximation is indeed rough, for $(3/15)a = 240$ cents is too large by 36.090 cents, $(2/15)a = 160$ cents is too small by 22.404 cents, and $(1/15)a = 80$ cents is too small by 31.731 cents.

A similar difficulty regarding the triad on II in the fifteen-note major scale is also found in the just scale, for perfect fifth DA is not pure; its frequency ratio is $40/27$, and it contains 680.449 cents. Neither is DF pure, for its frequency ratio is $32/27$, and it contains 294.135 cents. We may put DF and DA in pure tuning by using a D that is lower than the second-degree note by a syntonic comma (ratio $81/80$, size 21.506 cents). The two D's are sufficiently close together that they sound like two different versions of the same note; but this is not true in the case of the fifteen-note version. As we have seen, the distance between D‡ and D is 80 cents, which is the same size as the minor second; hence D‡ and D sound like two different notes. The greatly enlarged fifteen-

note version of the syntonic comma—nearly four times the interval's true size—is in large part responsible for the unacceptable disjunction occurring between II and V.

22. Successions of Triads Whose Roots Outline a Division of an Octave into Five Equal Parts

Since the major scale generated by three fifteen-note triads is not very good, we are led to wonder what other types of coherent organization of triads the tuning might furnish. Other possibilities are suggested by a closed circle of five perfect fifths (Section 20). If we arrange six major triads whose roots are successively C, F, B♭ ≡ A, D, G, and C (remembering that AB♭ is an enharmonic unison), so that parts move by the minimum possible distance, we have Example 24.



EXAMPLE 24

The same root succession may be agreeably harmonized if the triads are minor, or various alternations of major and minor. The entire array of possibilities is too numerous to list; however, while all are acceptable, some are better than others. As is so often the case regarding choices of this nature, the ear is the sole reliable guide.

An alien arrangement of triads that is both startling and immediately appealing results from placing a root-position major triad over each note that forms a descending five-note equal octave division, such as shown in Example 25.

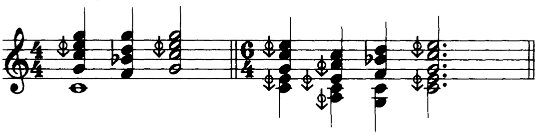
EXAMPLE 25

In similar progressions, triads may also ascend, modalities may be minor, or a combination of major and minor. Such progressions are remarkable in that they consist entirely of acceptably tuned triads; and yet they cannot in any way be realistically approximated on a piano. The essence of their singular nature is the underlying division of an octave into five equal parts.

23. Melodic and Harmonic Character of the Five-Note Equal Octave Division

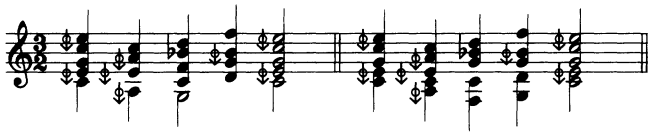
When played alone as an ascending or descending scale, the five-note equal division of an octave exhibits a musical character that is truly different from anything we are accustomed to. To conceive the sound of this scale without some means of actually hearing it is probably beyond the scope of human imagination. A rough idea may be got by observing that twelve-note tuning furnishes a division of an octave into three parts (an augmented triad), four parts (a diminished-seventh chord), or six parts (a whole-tone scale), but not five parts. The five-note equal division thus sounds like something midway between a diminished-seventh chord and a whole-tone scale. But it differs from the latter two divisions, for a diminished-seventh chord and a whole-tone scale are chromatic units, whereas the five-note equal division has a more diatonic character—at least, to my ear. The best subjective characterization I can find is that of a distorted pentatonic scale in which major seconds have become enlarged to the point that they are indistinguishable from minor thirds. In my experience, the five-note equal scale continues to sound agreeably startling after many repetitions over many years—I have never been able to get entirely accustomed to it. So much the more is true when the scale is harmonized by a succession of acceptably tuned triads, as in Example 25. This and similar arrangements of triads are about the most intriguing progressions this study has uncovered.

The five-note equal octave division is satisfactory when used as a chord in any inversion. In the recommended distributions, it sounds like a piquantly discordant, but weakly dissonant subdominant;[7] at present, I am unable to find a convincing theoretical explanation. When standing directly before the tonic triad, the best arrangements seem to be those that place the tonic or dominant note in the bass, as in Example 26.



EXAMPLE 26

When the five-note equal chord is a subdominant standing directly before a dominant seventh, the best arrangements place either the subdominant or the dominant note in the bass. The progressions in Example 27 are both good.



EXAMPLE 27

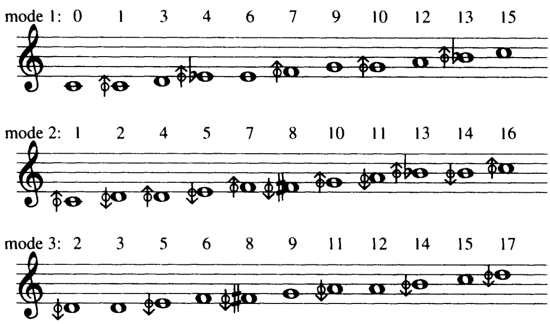
It will be observed that the progressions of Example 27 do not exhibit the same disturbing discontinuity that is associated with the progression II–V, as described in Section 21. The five-note equal chord is thus a valuable and useful subdominant, and may be expected to play an important role in the practical application of fifteen-note tuning. It is equally usable in those situations where the tonic triad is minor.

The symmetry exhibited by a five-note equal chord assures that any harmonic function ascribed to it with reference to a single tonic must also exist in exact transpositions with reference to four other tonics. From Example 26, we see that a five-note equal subdominant contains the tonic note, and hence the roots of the five possible resolutions are the same as the notes forming the subdominant chord itself. Thus the five triads whose roots outline a five-note equal octave division are closely linked through the five-note equal subdominant chord, its symmetry, and the closed circle of five perfect fifths. These elements taken together impart to fifteen-note equal tuning a modal character that cannot be realistically approximated in twelve-note tuning, and has not been hitherto explored.

24. The Ten-Note Symmetric Mode

It will often be convenient to regard fifteen-note equal tuning as three intertwined five-note equal octave divisions, each being $(1/15)a$ higher than the one just below. This is analogous to the vizualization of twelve-note tuning as three diminished-seventh chords, each higher than the one just below by $(1/12)a$, as described in Section 11. To pursue the analogy further, we may now define a ten-note symmetric mode (or decatonic scale) as a combination of two five-note equal divisions; this mode is an arrangement of ten notes within one octave in which adjacent intervals are alternately $(1/15)a = 80$ cents, and $(2/15)a = 160$ cents. Following the same line of reasoning as in Section 11, we

see that fifteen-note tuning contains three distinct ten-note modes; we shall call these modes 1, 2, and 3. The three modes are written out in Example 28, it being understood that there are other possibilities owing to enharmonics.



EXAMPLE 28

The distribution of accidentals makes it possible to recognize any one of these modes at a glance, for in mode 1, we find ↑'s (ups) affecting half the notes, but no ↓'s (downs); in mode 2, half the notes are affected by ↑'s and the other half by ↓'s; and in mode 3, we find ↓'s affecting half the notes, but no ↑'s. It can now be seen that Examples 22–27 are all entirely in mode 3. More generally, any major scale, or any succession of five major or minor triads whose roots outline either a circle of five perfect fifths or a five-note equal octave division, will be found within one of the ten-note symmetric modes. The fundamental diatonic elements of fifteen-note equal tuning are thus linked together through the ten-note symmetric mode in a manner that is unique among the tunings that will eventually be covered by this study.

The effect of the ten-note symmetric mode as a melody, either ascending or descending, is quite strange, owing partly to the fact that every other note outlines the alien five-note equal division, but more particularly to the use of $(2/15)a$ as a melodic interval (Section 18). In the absence of supporting harmonies, it is not especially pleasing—at least to my ear. But there are several good harmonizations that use only the ten notes forming the mode itself. For example, if the progression of Example 24 is continued similarly, so that each bar is higher than the preceding by $(1/5)a$, the superior part will outline mode 3, and all notes forming the triads will be found in the same mode, as can be seen at once from the distribution of accidentals.

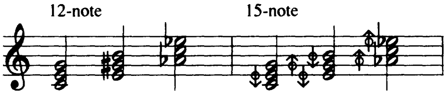
Another pleasing, if surprising arrangement harmonizes the mode with a succession of ten major triads in which notes of the superior part are alternately the third and fifth of each triad (Example 29).

EXAMPLE 29

There exists also a similar progression in which all the triads are minor; in this case, the notes needed will all be found in mode 1. There are also useful harmonizations of the mode that use all fifteen notes. One such arrangement uses the same distributions as Example 29, but with the triads alternately major and minor. Others will be described in Section 27.

25. The Six-Note Symmetric Mode

As we have seen, the symmetric modes of eight notes or ten notes may both be regarded as a division of an octave into an even number of intervals in which two different sizes alternate. Pursuing this idea further, we see that twelve-note tuning contains an array of six-note symmetric modes in which minor thirds and minor seconds alternate, e.g., C, E♭, E, G, G#≡A♭, B, and C. We may find a fifteen-note version of this same mode in positions 0, 4, 5, 9, 10, 14, and 15, corresponding to C, E♭⇞, E⇟, G, G⇞ ≡ A♭⇞, B⇟, and C. When heard as scales, the two versions are difficult to distinguish, and hence whatever triad progressions are associated with the six-note mode in twelve-note tuning may be expected to coexist in acceptable fifteen-note versions. In both versions, the mode links together three major triads whose roots outline an augmented triad, as shown in Example 30.



EXAMPLE 30

The three major triads associated with a single six-note mode exhibit familiar harmonic functions if they are arranged as in Example 31.

EXAMPLE 31

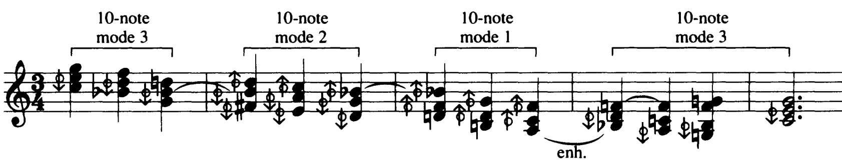
In the twelve-note version, the C triad is heard as a tonic, the A♭ triad as a subdominant, and the E triad as a dominant; the progression thus sounds like a chromatically altered version of I–IV–V–I. To my ear, the fifteen-note version produces exactly the same sensation of harmonic function and motion; the only palpable difference between the two is that the fifteen-note triads are more noticeably out of tune than their twelve-note counterparts.[8] If the triads of Example 31 are made minor instead of major, the resulting progression is still within the same six-note mode. In this case, however, the second chord does not sound like a subdominant, for its third is perceived as the leading tone relative to C, and this effectively destroys the subdominant sensation. Even so, this arrangement has proven useful in practical fifteen-note situations.

In the standard late-Romantic repertoire, we may occasionally find passages within a single six-note symmetric mode, although such passages are far less common than octatonic passages.[9] Perhaps the reason in part is that no very good melody may be made from a single six-note mode, for the mode contains no major seconds—and very rare indeed is the good melody that totally avoids major seconds. This suggests that the six-note mode is less useful as an organizing force in fifteen-note tuning than the ten-note mode.

26. Progressions Involving an Alternation of Six-Note and Ten-Note Modes

One especially useful aspect of the six-note symmetric modes of fifteen-note tuning is that they make possible a smooth modulation from one ten-note mode to either of the other two contained by the tuning. Referring to the fifteen-note version of Example 30, distribution of accidentals shows at once that the C major triad is found in ten-note mode 3 (Example 28, Section 24), the E⇓ major triad in ten-note mode 2, and the A♭⇑ major triad in ten-note mode 1. A similar situation will be found to hold regarding minor triads. Thus given any passage within one of the ten-note modes, we may stop at any triad,

find this triad in one of the six-note modes, and then use the relation established by the six-note mode to modulate into either of the other two ten-note modes. This opens the way to a large class of progressions of triads in which the six-note and ten-note modes alternate, sometimes in a sequential or cyclic manner. In general, at least three harmonies are needed to establish identifiable relations associated with the ten-note mode; in the case of the six-note mode, two chords will suffice. An especially pleasing progression in this category is shown in Example 32.



EXAMPLE 32

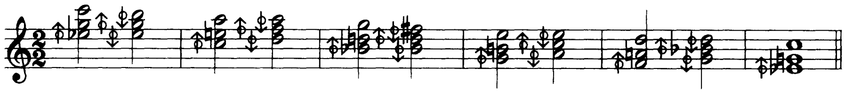
It must not be thought that any succession of triads within fifteen-note equal tuning will be harmonically coherent; in the absence of any symmetry, cyclic elements, or identifiable modality, progressions seem to wander about without direction.

In general, progressions of the sort illustrated in Example 32 sound more intricate than those associated with a single ten-note mode or a single six-note mode. In my opinion, they should be used rather infrequently in a style where diatonic relations predominate. However when played slowly with elaborate ornamentations, they may serve as an underpinning to larger musical periods or forms.

27. Other Cyclic Progressions

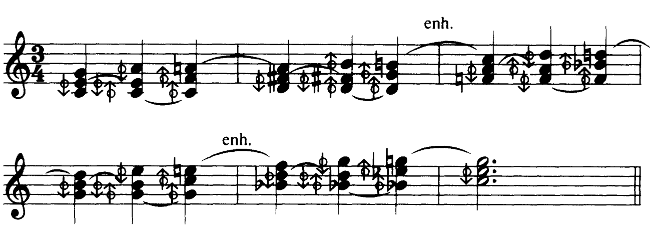
Another useful class of chromatic progressions may be found by starting with a pair of triads, one major and one minor, within a single six-note mode, and using this to establish a modulating sequence whose interval of transposition is $(\frac{1}{5})a$. For example, C minor and E↓ major use all the notes of mode C, E♭↑, E↓, G, G↑ ≡ A♭↑, B↓, and C (Section 25); if these are the first two triads, the sequence is as shown in Example 33. It will be observed that the superior part outlines ten-note mode 3 (Example 28, Section 24).

We may find another useful cyclic arrangement of major triads in which the third of each triad becomes the fifth of the following triad. With the arrival



EXAMPLE 33

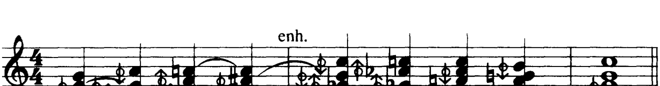
of each successive triad, the root falls a minor third of $(\frac{4}{15})a = 320$ cents. The progression uses all fifteen triads, and furnishes yet another harmonization of ten-note mode 3 (Example 34). The reader will find it interesting to observe how ten-note mode 3 is presented in different enharmonic spellings in Examples 29, 32, and 33, depending on context.



EXAMPLE 34

The chromatic progressions of Examples 32–34 do not by any means exhaust the subject; they were chosen as particularly representative members of a much larger class of progressions.

Another useful aspect of more intricate progressions is how they may be used to establish special chromatic relations that connect the primary triads of a major key. Note that in Example 34 the roots of the first and seventh triads are C and F, respectively; this establishes a chromatic relation between a tonic and its subdominant. This may be revealed by playing in order the first seven triads of Example 34, and then a standard IV–V^7–I cadence (Example 35).



EXAMPLE 35

It should now be recalled that another version of this same progression, having identical harmonic behavior but slightly different intonation, may be

found in nineteen-note equal tuning, as illustrated in Example 5 (Section 5). We thus have a fascinating example of a progression which exists in recognizable versions in both fifteen-note and nineteen-note tuning, but cannot be found within the notes of twelve-note tuning.

28. Comparison of Fifteen-Note and Sixteen-Note Equal Tunings

When fifteen-note and sixteen-note tunings are played in ascending or descending order, they sound like compressed chromatic scales; since adjacent intervals are so nearly equal (80 cents and 75 cents, respectively), the two scales can be distinguished only if one counts carefully. Another point of similarity is the difficulty associated with melodic intervals $(2/15)a$ and $(2/16)a$, which has proven to be very substantial in practical situations. This shows that fifteen-note and sixteen-note tunings have a rather similar melodic character.

Harmonically, however, fifteen-note and sixteen-note tunings are entirely different. Elements organized are consonant triads in fifteen-note tuning, and dissonant seventh and altered chords in sixteen-note tuning. But more importantly, fifteen-note tuning contains a much broader variety of tonal resources than sixteen-note tuning. This is associated with the contrasting ten-note and six-note modes of fifteen-note tuning, the former containing several distinct substructures, as opposed to the single octatonic modal arrangement found in sixteen-note tuning.

In sum, fifteen-note equal tuning possesses a variety of beautiful and fascinating harmonic forces within a tonal framework consisting mainly of triads. It must be conceded that modern musicans will find it out of tune, but not, I think, excessively so. It is, of course, useless for any of the existing repertoire.

The equal tuning of fifteen notes is readily adaptable to certain classes of conventional instruments. In particular, guitars and other fretted stringed instruments may be easily modified—all that is needed is a redesigned fret board. Open strings are then tuned E♮↓, A♮↓, D♮↓, G♮↓, B♮↓≡C♮↓, and E♮↓, eliminating the single major third (GB) among the perfect fourths that is a feature of standard twelve-note guitar tuning, with a corresponding simplification in fingering.

Earlier theorists have generally held that fifteen-note equal tuning is of little or no practical use; with this opinion, I am in complete disagreement. On the basis of my now extensive practical experience, I am persuaded that fifteen-note equal tuning is likely to bring about a considerable enrichment of both classical and popular repertoire in a wide variety of styles.

NOTES

1. Easley Blackwood, *The Structure of Recognizable Diatonic Tunings* (Princeton: Princeton University Press, 1985), 30, 160, 164, 210. This work will henceforth be referred to as *Structure*.

2. The symbol "≡" means "is enharmonically equivalent to."

3. Since the early nineteenth century, it has been fairly common for the development sections of sonata forms to modulate all the way around the circle of twelve fifths, resulting in a complete enharmonic shift. In the first movement of Beethoven's Piano Sonata, Op. 57, there are two such shifts; the succession of keys is F minor, A♭≡G# minor, E major, E minor, C minor, A♭ major, D♭ major, B♭ minor, G♭≡F# major, B minor, C major, and F minor.

4. *Structure*, 311.

5. Ibid. 226–28.

6. Ibid. 69–70.

7. The reader is reminded of the distinction drawn between dissonance and discordance, as described in Section 10.

8. The progression of Example 31 may be found in the standard repertoire. See the first two bars of the second movement from the second suite of Prokofiev's *Romeo and Juliet* (Juliet—The Little Girl). A transposed version serves as a harmonization of chromatic scales in Liszt's Piano Concerto in E♭. See the first movement, bars 65–73; another transposition occurs in the second movement, bars 182–90.

9. See the first twenty-four bars of Rimsky-Korsakov's symphonic poem *Antar* (in the 1875 version); Rachmaninov's Etude Op. 39, No. 2, bars 68–71, and Prokofiev's Toccata Op. 11, bars 111–18.